E80 S'OS Thust & Kinetics Lecture

What is Newton II?

Usually written as

\[ \sum \vec{F} = m \vec{a} \]

Oversimplification

\[ \sum \vec{F} = \frac{d}{dt} (m \vec{v}) = m \vec{a} + m \vec{\dot{a}} \]

For constant \( m \), we get usual.

What makes rockets fly?

High velocity substances exiting out the back.

At steady state \( m \) and \( \vec{v} \) of the gases are constant so

\[ \sum \vec{F} = m \vec{v} \] at steady state

\[ F = m \vec{v} \quad 1-D \]

For the transient

\[ F = \frac{d}{dt} (m \vec{v}) \quad 1-D \]
Note: The mass of the rocket is changing.

Where do you get the hot gases? Usually from combustion or oxidation reactions.

Reaction kinetics

For solid propellant - heterogeneous R.K.

Before we go there, we need some fluid mechanics and some thermo.

It can be shown for an ideal gas that

\[ C_v dT = T d\hat{S} - P d\hat{V} \]

\( C_v \) - heat capacity at constant volume

\( T \) - temperature (\( \text{abs} \))

\( \hat{S} \) - specific entropy

\( P \) - absolute pressure

\( \hat{V} \) - specific volume
\[ d\hat{S} = \frac{C_v}{T} dT + \frac{R}{T} \hat{V} \]

OR

\[ d\hat{S} = \frac{C_p}{T} dT - \frac{R}{p} dp \]

- \( C_p \) - heat capacity at const. \( p \)
- \( R \) - universal gas constant = \( N_a k \)

If we design our nozzle carefully,
\[ d\hat{S} = 0 \] or \( \Delta \hat{S} = 0 \) Adiabatic Reversible

\[ \frac{C_p}{T} dT = \frac{R}{p} dp \]

If \( C_p \neq C_p(T) \) Never true, usually acceptable

\[ \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^\gamma = \left(\frac{P_2}{P_1}\right)^\left[\frac{1}{\gamma - 1}\right] \]

\[ \gamma = \frac{C_p}{C_v} \]

If we analyze the nozzle with 1st Law
\[ \Delta H + \Delta E_k = \Delta Q - \Delta W_s \]

\[ m (\hat{v}_2 - \hat{v}_1) + \frac{1}{2} m (v_2^2 - v_1^2) = 0 \]

\[ \hat{v}_2 - \hat{v}_1 = c_p (T_2 - T_1) \]

\[ v_2^2 - v_1^2 = 2 c_p (T_2 - T_1) = 2 c_p T_1 \left[ 1 - \left( \frac{P_2}{P_1} \right)^{\frac{1}{\gamma}} \right] \]

There are equivalent expressions. Be very careful with units.

\[ \nu_e = \left( \frac{RT_1}{m_w} \right)^{\frac{2 \gamma}{\gamma - 1}} \left[ 1 - \left( \frac{P_e}{P_i} \right)^{-\frac{1}{\gamma}} \right]^{\frac{1}{2}} \]

\[ m_w - \text{gas molecular weight,} \quad \frac{k_3}{\text{kmol}} \text{ for SI} \]

The de Laval nozzle is sonic in the throat and supersonic beyond. Take compressible flow to see why.

A plot of \( F_{\text{thrust}} \) vs \( t \) is a plot of \( \frac{d(mv)}{dt} \) vs \( t \). \( m, v_e \)

* if nozzle is not ideal and exit pressure \( P_e = P_{\text{amb}} \), there is an \( Pe A_e \) additional term on the thrust.
Chemical Kinetics

For homogeneous reactions

\[ \frac{[\text{mol}]}{[\text{m}^2 \cdot \text{s}]} = \frac{1}{V_i} \cdot \frac{d[N_i]}{dt} \]

\( n_i \) - moles of i

\( V_i \) - stoic coeff.

For heterogeneous reactions

\[ \Gamma_s = \frac{1}{V_i} \cdot \frac{d[N_i]}{dt} \]

\( s \) - surface area

Possible steps

1) diffuse oxidizer to surface
2) adsorb on surface
3) react on surface
4) desorb from surface
5) diffuse product from surface.

Additional complications

Diffuse through bulk
Surface may evaporate and react in gas phase.
In our case

\[ 2 \text{NH}_4\text{ClO}_4(s) \rightarrow 2\text{O}_2(s) + \text{N}_2(s) + 4\text{H}_2\text{O}(s) + 2\text{Cl}_2(s) \quad (1) \]

\[ 4\text{Al}(s) + 3\text{O}_2(s) \rightarrow 2\text{Al}_2\text{O}_3(s) \quad (2) \]

\[ 2\text{Al}(s) + 3\text{Cl}_2(g) \rightarrow 2\text{AlCl}_3(g) \quad (3) \]

Holding everything together is HTPB (Hydroxyl-terminated polybutadiene).
It also burns

\[ \text{HO(C}_4\text{H}_4)_{n}\text{OH} + \text{O}_2(s) \rightarrow \text{CO}_2(s) + \text{H}_2\text{O}(g) \]

(Not balanced)

A complete analysis of the kinetics is incredibly involved.
We're going to ignore lots of details and assume

\[ r = kP^n \]

The reasons are subtle but involve assuming that the surface rate is so fast that the surface and the
gas just above it are in equilibrium at a pressure, $P$, and that as $P_1$ (in the motor) changes $T_{exh}$ changes so $kP_m$ is adequate.

We can't measure $P_1$, $T_1$, or $S(t)$
All we can measure is $S_0$ and $F_{thrust}$.

We don't know the exact mixture of $NH_4ClO_4$, $Al$, and HTPB but we can guess at the ratios.
We don't know $m_0$ or $v_0$ independently.
What can we do? YMMV

D Weigh before and after. Then we know the integral of $m$.

2) Assume some relationship between $m$ and $v_e$
   e.g. $m v_e$.

3) Assume an initial and burn geometry.
   e.g. cylinder
Regress the data

Example: Assume propellant is hollow cylinder of total constant length.

At any instant $A = 2\pi r L$

$\begin{align*}
\frac{dV}{dt} &= 2\pi r L \frac{dr}{dt} \\
\frac{dm}{dt} &= \rho \frac{dV}{dt}
\end{align*}$

$\frac{\rho}{\rho} \frac{dV}{dt} = (kP^n) (2\pi r L) \frac{dr}{dt}$

$\rho (2\pi r L) \frac{dr}{dt} = kP^n (2\pi r L)$

$dr = \frac{kP^n}{\rho} \frac{dt}{t}$

$t - t_0 = \frac{kP^n}{\rho} t$

$\begin{align*}
\dot{m} &= 2\pi L kP^n \left( \frac{kP^n}{\rho} t - t_0 \right) \\
\dot{m} &= 2\pi L kP^n \frac{t}{\rho} - 2\pi L kP^n t_0
\end{align*}$
Fitting a straight line to \( \dot{m} \) vs. \( t \) should give \( kP^n \) from the slope and intercept.

Alternately,

\[
kP^n = \frac{\pi L r_0 \rho + \dot{m} 2\pi L r_0 \rho + (\pi r_0 \rho L)^2}{2\pi L t}
\]

(double check method)

Can plot \( kP^n \) vs. \( t \) and see if it makes sense.

Goals for experiment

1) Get thrust curves

2) See if you can regress some kinetic parameters

3) Compare kinetic parameters for different motors