Thrust and Flight Modeling
Static Motor Tests and Flight Modeling Lab

http://twistedsifter.com/2012/10/red-bull-stratos-space-jump-photos/
First Flight

BEM
Outline

- Static Motor Tests and Flight Modeling Lab Overview

- Flight Modeling:
  - 1 Degree of Freedom (DOF) Model
  - 3 DOF Model
Static Motor Rotation Lab Objectives

• Measure the thrust curves, mass flow rate of combustion gases and specific impulse for two rocket motors.

• Construct analytical and 1-D (1 DOF) and 2-D (3 DOF) numerical models of rocket flight.

• Compare the analytical and numerical models with the output of RockSim or OpenRocket.
Where will the rocket go?
Flight modeling

- What key forces dictate the flight trajectory?
  - Thrust, Drag, Gravity, Lift
Reminder: Lift and Drag

The sum of pressure and shear stress is the resultant force. It is split into two components:

1. **Lift**: The component of resultant force that is *perpendicular* to the *incoming net velocity vector* (effective flow direction).

2. **Drag**: The component of resultant force that is *parallel* to the *incoming net velocity vector* (effective flow direction).
One DOF Model: Free Body Diagram
One DOF Model: Governing Equation

\[ F = m \ddot{z} = T - mg - F_D \]

\[ m = m(t) \]
Modeling Thrust

- Is thrust constant during flight? No. $T(t)$
Experimental Engineering

http://v-serv.com/usr/estesmotors.htm
Static Motor Rotation Lab

- O-rings
- Propellant grains
- Nozzle
- Delay grain
Static Motor Thrust Curve

http://www.eng.hmc.edu/NewE80/StaticTestVideos.html
Static Motor Lab, Section 2:

• Calculate the total impulse.
• Calculate the average thrust and average mass flow rate.
• Calculate the exit velocity of the combustion gases from the nozzle. What assumptions did you have to make?
• Calculate the specific impulse, \( I_{sp} \).
Drag Force

\[ F_D = \frac{1}{2} \rho V^2 A C_D \]

\[ A = \text{reference area} \]

\[ C_D = f(Re, \alpha) \]

\[ \alpha = \text{angle of attack} \]
Analytical One DOF Model

• GE:

$$m\ddot{z} = T - mg - F_D$$

• Assumptions:
  - \(\text{Const } m\)
  - Constant \(T\) (over a time interval)
  - Constant \(C_D\)
Numerical One DOF Model

• **GE:**

\[ m(t) \ddot{z} = T(t) - m(t) g - \frac{1}{2} \left( \dot{z}^2 A C_D \right) \]

• Many options for numerical solution methods, e.g.
  - OpenRocket uses **Runge-Kutta** (RK4)
  - One option is **Explicit Euler** ignoring high order terms...
Explicit Euler

\[ y(t) = f(y, t) \]

For each time step of size \( h \),

\[ y_{n+1} = y_n + hf(y_n, t_n) \]

\[ t_{n+1} = t_n + h \]
One DOF Model

for t = 0 to maxTime
{
    T = ...
    m = ...
    Fd = ...

    \[ z_{dd}(t) = \frac{1}{m}(T-mg-Fd); \]
    \[ z_d(t) = z_d + z_{dd}\Delta t \]
    \[ z(t) = z + z_d\Delta t \]
}
Three DOF Model

- What are the 3DOF?
Why does the rocket rotate?

- **Initial direction of motion**
- **New Direction of Motion**

**Wind**

- **Direction of flow from rocket motion**
- **Effective flow direction**
Reminder: Angle of Attack

- Chord Line
- Angle of Attack
- Relative Wind
- Lift
- Resultant Force
- Drag
- Center of Pressure
Angle of Attack

\[ \alpha = \theta - \gamma \]
Three DOF Free Body Diagram
Non-Rotational Forces

- **z-direction**
  \[
  m\ddot{z} = T \sin \theta - mg - F_d \sin \theta + F_L \cos \theta
  \]

- **x-direction**
  \[
  m\ddot{x} = T \cos \theta - F_d \cos \theta - F_L \sin \theta
  \]
Torque Balance

\[ I \ddot{\theta} = -T_D - r F_L \]
Rotational Damping

- The rotational damping can be modeled as

\[ T_b = C \dot{\theta} \]

\[ C = \text{damping coefficient} \]
Rocket Stability

- Is this stable?
  - Depends on location of $C_P$ versus $C_G$
Reminder: Complication #3 Angle of Attack

$$C_L, C_D = f(\alpha, \text{Re})$$
Drag and Lift direction

- Drag and lift can be defined w.r.t.

Rocket axis

Effective flow direction
Three DOF Model

for \( t = 0 \) to maxTime
{
    \( T = \ldots \),
    \( m = \ldots \),
    \( Fd = \ldots \),
    \( L = \ldots \),
    \( Td = \ldots \),
    \( \alpha = \ldots \),

    \( z_{\text{dd}}(t) = \ldots \),
    \( x_{\text{dd}}(t) = \ldots \),
    \( \theta_{\text{dd}}(t) = \ldots \),
    \( \ldots \)
}
To Linde Field

• Good luck!