ESTIMATION AND ANALYSIS OF QUASI-1D SOLID ROCKET MOTOR

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INCLUDED TOPICS:

1. THERMODYNAMICS
2. CHEMICAL KINETICS
3. THRUST ANALYSIS
4. IMPULSE ANALYSIS

1. THERMODYNAMICS

Thermodynamic entropy is defined as (in differential form):

$$dS \equiv \frac{\delta Q_{rev}}{T} \quad (1.1)$$

Because flow through a nozzle is a reversible process it is isentropic. The change in internal energy (*E*), pressure (*P*), enthalpy (*H*), and volume (*V*) are related to the heat flux by energy conservation. This can be represented in differential form:

$$dQ = dE + PdV = dH - VdP = 0 \quad (1.2)$$

Because the change in entropy is zero, the heat transfer must be zero. This leads to the following thermodynamic relationships:

$$dE = C_v dT \quad dH = C_p dT \quad (1.3)$$

Where C_v and C_p are the heat capacities at constant volume and pressure, respectively. Substituting *1.3* into *1.2* and dividing by temperature:

$$dS = \frac{\delta Q}{T} = C_v \frac{dT}{T} + R\frac{dV}{V} = C_p \frac{dT}{T} - R\frac{dP}{P} = 0 \quad (1.4)$$

1.4 can be integrated to yield an explicit solution evaluated between points 1 and 2:

$$S_2 - S_1 = C_v \ln\frac{T_2}{T_1} + R\ln\frac{V_2}{V_1} = C_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = 0 \quad (1.5)$$

The thermodynamic heat ratio (γ) and gas constant (R) are related to C_p and C_v:

$$\gamma = \frac{C_p}{C_v} \qquad R = C_p - C_v \qquad \overline{R} = \frac{R}{\mathcal{M}} = \frac{C_p - C_v}{\mathcal{M}} \quad (1.6)$$

For rocket motor combustion products, $\gamma \approx 1.20$. With algebraic manipulation we can find temperature dependence on pressure:

$$\ln\frac{T_2}{T_1} = \frac{R}{C_p}\ln\frac{P_2}{P_1} \rightarrow \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{R}{C_p}} = \left(\frac{P_2}{P_1}\right)^{1-1/\gamma} \qquad (1.7)$$

The local sonic velocity (a, m/s) can be found by:

$$a = \sqrt{\gamma \bar{R} T} \qquad (1.8)$$

Where $\bar{R}$ is the specific gas constant for the local gas and has units J/kg-C. The Mach number (*M*) at a point describes a velocity relative to its local sonic speed. This is defined to be:

$$M \equiv \frac{v}{a} = \frac{v}{\sqrt{\gamma \bar{R} T}} \qquad (1.9)$$

Suppose we wish to examine the energy of two distinct points in a flow. Perhaps first point has a very low velocity such that the great majority of its energy is in the form of heat. The corresponding stagnation temperature T_o and specific heat capacity c_p must equal the energy present at some point x in the form of thermal and kinetic energy. Therefore, if flow losses are negligible, energy continuity must apply:

$$c_p T_o = c_p T_x + \frac{{v_x}^2}{2} \qquad (1.10)$$

By rearranging *1.10* and applying *1.9* we can find the relationship between Mach number and the thermal gradient:

$$\frac{T_o}{T_x} = 1 + \frac{v_x^2}{2c_p T} = 1 + \frac{\gamma - 1}{2}\frac{v_x^2}{\gamma \bar{R} T} = 1 + \frac{\gamma - 1}{2}{M_x}^2 \qquad (1.11)$$

This thermodynamic relationship can be used to find pressure dependence on Mach number between two distinct points *1* and *2*. Because we wish to apply this to rocket nozzle kinetics, we will denote point *1* as the stagnation point as found in the combustion chamber of the rocket motor. Point *2* can be any point "downstream" though for practical purposes we define it to be the exit of the nozzle.

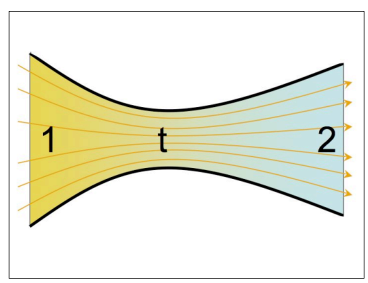

Figure 1. Flow through a De Laval Nozzle

$$\frac{T_1}{T_2} = 1 + \frac{\gamma - 1}{2}{M_2}^2 = \left(\frac{P_1}{P_2}\right)^{1-1/\gamma} \qquad (1.12)$$

Rearranging *1.12* and solving for pressure, the relationship between the exit Mach number and the pressure ratio can be computed:

$$\frac{P_1}{P_2} = \left(1 + \frac{\gamma - 1}{2} M_2^{\ 2}\right)^{\frac{\gamma}{\gamma-1}} \quad (1.13)$$

This can be used to solve for M_2:

$$M_2 = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{P_1}{P_2}\right)^{1-1/\gamma} - 1\right]} \quad (1.14)$$

The exhaust velocity can be easily related to the in terms of the exit Mach number:

$$v_e = v_2 = M_2\sqrt{\gamma \bar{R} T_2} \quad (1.15)$$

2. CHEMICAL KINETICS

The propellant used in the E80 motors is a variety of Ammonium Perchlorate Composite Propellant (APCP). It is an excellent solid rocket fuel, very similar to the formulation used on the Space Shuttle's Solid Rocket Boosters (SRBs). A mixture of ammonium perchlorate (AP) and powdered aluminum (Al) is intimately suspended in hydroxyl-terminated polybutadiene (HTPB) rubber. The AP acts as the oxidizer while the Al and HTPB act as the fuel. Rough proportions for a basic APCP propellant formulation is 70/15/15 of AP/Al/HTPB. Small amounts of additives (for curing and reaction catalysts) also exist in APCP and can play a crucial role on the burn characteristics. Another burn-characteristic modifier is the size of the AP and Al particles used in the propellant, though this is beyond the scope of E80.

APCP has an energy density roughly three times that of gunpowder though it does not detonate or operate like gunpowder. The chemical process is *deflagration* – a stable surface-burning processes in which propellant particles are heated (or vaporized) on the surface due to ambient heat until they react and produce gas, revealing a layer of unburned material underneath that undergoes the same process. Because of this, the initial geometry of the fuel grain changes and therefore (in nearly all cases) the propellant surface area (*A*) changes.

The first step in combustion of APCP is the thermal decomposition of ammonium perchlorate:

$$2NH_4ClO_4(s) + heat \rightarrow 2O_2(g) + N_2(g) + 4H_2O(g) + Cl_2(g) \quad (2.1)$$

When sufficient heat is added the aluminum fuel combusts:

$$4Al(s) + 3O_2(g) \rightarrow 2Al_2O_3(s) + heat \quad (2.2)$$

There is also a reaction between aluminum and chlorine gas:

$$2Al(s) + 3Cl_2(g) \rightarrow 2AlCl_3(g) + heat \quad (2.3)$$

The net reaction between oxidizer and aluminum can be summarized as:

$$6NH_4ClO_4(s) + 10Al(s) \rightarrow 4Al_2O_3(s) + 2AlCl_3(g) + 3N_2(g) + 12H_20(g) + heat \quad (2.4)$$

The thermal decomposition of HTPB can be approximated by:

$$HO\left(C_4H_4\right)_{50}OH + heat \rightarrow 49C_4H_4(g) + 2CO(g) + C_2H_6(g) \quad (2.5)$$

Combustion of HTPB decomposition products:

$$C_4H_4(g) + 5O_2(g) \rightarrow 4CO_2(g) + 2H_2O(g) + heat \quad (2.6)$$

Because the chemical kinetics of solid rocket fuel combustion are very complex, it has been demonstrated that a good reaction-rate model can be expressed as a simple exponential relation between the linear burn rate (*r,* m/s), the pressure exponent (*n,* m/s), the non-dimensional combustion pressure ($\overline{P}$) and the pressure coefficient (*k*):

$$r = k\overline{P}^n \quad (2.7)$$

There is also a rate dependence on temperature though this is beyond the scope of this class and can be neglected in rough models.

Given a known amount of surface area (*A,* m^2) reacting in the combustion and the average density of the APCP (ρ,kg/m^3), the mass flux through the nozzle is expressed by:

$$\dot{m} = \rho \dot{V} = \rho A r \quad (2.8)$$

3. Thrust Analysis

Conservation of linear momentum results in a net system force.
The majority of thrust comes from the high velocity mass flux through the nozzle:

$$\vec{F}_{flux} = \dot{m}\vec{v}_e \quad (3.1)$$

However, a second contribution of thrust comes from the fact that the combustion chamber operates at a given pressure and this pressure is exerted on all walls, though since the nozzle throat is open, there is a net force from the pressure of the combustion against the front end of the combustion chamber as projected from the cross-section of the nozzle throat. This pressure gradient contributes to the linear momentum of the exhaust gases:

$$\vec{F}_{pressure} = \Delta P_{ea}\vec{A}_e \quad (3.2)$$

where $\Delta P_{ea} = P_e - P_a$.

In general form (for N nozzles or holes in the motor):

$$\vec{F} = \sum_{i}^{N} \left[\frac{d}{dt}\left(m\vec{v}\right)_i + \left(\Delta P_{ea}\vec{A}_e\right)_i \right] \qquad (3.3)$$

In a De Laval nozzle, the net thrust produced can be found by:

$$F = F_{flux} + F_{pressure} = \dot{m}v_e + \Delta P_{ea}A_e = \dot{m}v_{eq} \quad (3.4)$$

where A_e is the diameter of the nozzle throat and P_a is the ambient pressure. (Note that Aerotech nozzles may be slightly under-expanded.)

4. Impulse Analysis

When testing a rocket motor/engine as a whole, it can be useful to look at the performance of the system by including the propellant utilization rather than only the nozzle characteristics. The total impulse (I, N-s) of a rocket motor/engine is defined as:

$$I_{total} \equiv \int_0^T \left|\vec{F}(t)\right| dt \qquad (4.1)$$

For "F" and "G" class motors, the impulse will be roughly between 50-60 Ns and 100-120 Ns, respectively. The total impulse and used propellant weight during burn of duration T can be used to calculate the specific impulse (I_{sp}, sec).

$$I_{sp} = \frac{I_{total}}{g\Delta m_{motor}} = \frac{v_{eq}}{g} \qquad (4.2)$$

The specific impulse is a useful parameter when comparing the efficiency of two or more propulsion systems. For the APCP motors used in E80, the specific impulse will roughly be in the range of 180-215 sec.