Vibration and System Identification

(Adapted from previous years’ lectures)

Prof. Angie Lee
### Remaining lectures

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Start thinking about final project!
Folsom Dam: vibration testing

Shaker: sinusoidal input

http://itll.colorado.edu/test_measurement_equipment/vibration_testing/

Cold gas thruster: impulse input
• Interesting examples of rocket vibrations
• Intro to system identification and modal vibration
• Vibration analysis (math model, computational model)
• Vibration testing (experiment)
• Application: cantilever beam
• Application: rocket
• Fun video
Example of rocket vibration

- HEAT1X-Tycho Brahe inaugural flight
- Pilot's POV – 9 Hz oscillation
- [http://www.youtube.com/watch?v=-rASHRBo9Rg&feature=player_embedded](http://www.youtube.com/watch?v=-rASHRBo9Rg&feature=player_embedded)
“Pain was directly associated with motion of the eyeballs and testicles, as well as from internal heating that resulted from sloshing of the brain and viscera. The vibration frequency was also in the range of normal brain waves, adding confusion to decision making, hand and arm movement, and even speech.”

Jim Fenwich on Pogo oscillations
Example of rocket vibration

Space shuttle main engine turbopumps

“The high-pressure pumps rotated at speeds reaching 36,000 rpm on the fuel side and 24,000 rpm on the oxidizer side. At these speeds, minor faults were exacerbated and could rapidly propagate to catastrophic engine failure.”

“…the vibration spectral data contained potential failure indicators in the form of discrete rotordynamic spectral signatures. These signatures were prime indicators of turbomachinery health…”

"Wings in Orbit" edited by Wayne Hale and Helen Lane
“While the lower stages of the North Korean rocket continued to function for several minutes, resonance at the top of the launch vehicle resulted in ‘catastrophic disassembly' of the third stage at Max Q,’ said Charles Vick, senior technical and space policy analyst at GlobalSecurity.org. ‘The vibrations just tore it apart.’”


Causes of rocket vibration

- Thrust oscillations
- Noise (pressure waves) due to motor or engine
- Fluid flow phenomena (aerodynamic stress)
  - Wind
  - Turbulence
  - Vortex shedding
Question

Why might we be interested in rocket vibrations?
• System identification:
  building mathematical models of dynamical systems based on observed input-output data

• Modal analysis:
  characterization of vibrational mode shapes and corresponding frequencies of a physical system

• Dynamic load and response:
  load (input) applied dynamically (varying over time)
  the response associated with this load is the dynamic response of the system

BASIC IDEA/GOAL
Determine the modal properties of a system:
natural frequencies and modal shapes
How do you approach the problem of analyzing a structure with dynamic loading?

(a) Model your physical problem
   Geometry, kinematics, material, loading

(b) Derive governing equations (mostly differential equations)

(c) Solve the equations

(d) Interpret results and refine and repeat!

What else can you do to characterize a structure?

Take measurements!

Experimental studies to validate a model or help develop a model
A simple model: spring-mass-damper system

- Around a resonance frequency, you can model as

\[ m_e \ddot{y} = f - ky - cy \]
\[ m \ddot{y} + c \dot{y} + ky = f \]
\[ \ddot{y} + \frac{c}{m_e} \dot{y} + \frac{k}{m_e} y = \frac{f}{m_e} \]
\[ \ddot{y} + 2\zeta \omega_n \dot{y} + \omega_n^2 y = \frac{f}{m_e} \]

\[ \omega_n = \sqrt{\frac{k}{m_e}} \]
\[ \zeta = \frac{c}{2\sqrt{m_ek}} \]
Frequency response function (FRF)

- Position

\[
\frac{Y}{F} = \frac{1}{m_e} \left( \frac{1}{\omega_n} \right)^2 \frac{1}{1 - \left( \frac{\omega}{\omega_n} \right)^2 + 2\zeta \frac{\omega}{\omega_n} j}
\]

- Velocity

\[
\frac{V}{F} = \frac{j\omega}{m_e} \left( \frac{1}{\omega_n} \right)^2 \frac{1}{1 - \left( \frac{\omega}{\omega_n} \right)^2 + 2\zeta \frac{\omega}{\omega_n} j}
\]
Frequency response function (FRF)

- Acceleration

\[ \frac{A}{F} = -\frac{1}{m_e} \left( \frac{\omega}{\omega_n} \right)^2 \left( 1 - \left( \frac{\omega}{\omega_n} \right)^2 + 2\zeta \frac{\omega}{\omega_n} j \right) \]
Damping coefficient

- From the peak \( \omega_r = \omega_n \sqrt{1 - \zeta^2} \)
- From the half-power bandwidth \( \Delta \omega = \omega_{+hp} - \omega_{-hp} \)

\[
Q = \frac{\omega_r}{\Delta \omega}
\]

\[
\zeta = \frac{1}{2Q}
\]

http://www.sengpielaudio.com/calculator-cutoffFrequencies.htm
Beams

- We can extend this idea from the spring-mass-damper example to a more complicated structural element, such as a beam.

- Beams are one of the most important components in structural engineering.

  - Examples of beams: bridges, walkways, rockets, …

https://en.wikipedia.org/wiki/Beam_(structure)
Some important properties and characteristics of a beam are:

1) Cross sectional area: $A, I$
2) Length: $L$
3) Material: $E, \rho$
4) Supports (boundary conditions)

<table>
<thead>
<tr>
<th>Type</th>
<th>Boundary Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pinned or hinged end</td>
<td>Left end: $w(0) = 0, -EI \frac{d^2w(0)}{dx^2} = 0$ Right end: $w(L) = 0, EI \frac{d^2w(L)}{dx^2} = 0$</td>
</tr>
<tr>
<td>Clamped or fixed end</td>
<td>Left end: $w(0) = 0, \frac{dw(0)}{dx} = 0$ Right end: $w(L) = 0, \frac{dw(L)}{dx} = 0$</td>
</tr>
<tr>
<td>Free end</td>
<td>Left end: $-EI \frac{d^2w(0)}{dx^2} = 0, EI \frac{d^2w(L)}{dx^2} = 0$ Right end: $EI \frac{d^2w(L)}{dx^2} = 0, -EI \frac{d^2w(L)}{dx^2} = 0$</td>
</tr>
<tr>
<td>Sliding or guided end</td>
<td>Left end: $\frac{dw(0)}{dx} = 0, -EI \frac{d^2w(0)}{dx^2} = 0$ Right end: $\frac{dw(L)}{dx} = 0, -EI \frac{d^2w(L)}{dx^2} = 0$</td>
</tr>
</tbody>
</table>

http://cnx.org/contents/mu6YpDEl@1/Beams-pillarsstruts-crossbars-
Experiments: vibration testing

- Lab tests
  - Shaker tests
    https://www.youtube.com/watch?v=o8H_NT7Ziao

- Impact hammer tests
  https://www.youtube.com/watch?v=tBRjPN8m6zE
Cantilever beam (rotation lab)

• Mathematical model
  to obtain natural frequencies and modal shapes

• Computational model
  example: SolidWorks model

• Experimental data
  tap test in lab using strain gauges
Cantilever vibration modes

Fig. 4.1 (a): A cantilever beam

Fig. 4.1 (b): The beam under free vibration

Fig. 4.3: The first three undamped natural frequencies and mode shape of cantilever beam

http://iitg.vlab.co.in/?sub=62&brch=175&sim=1080&cnt=1

https://www.youtube.com/watch?v=kun62B7VUg8

E80 Lecture 10.21: Vibration and System ID
Cantilever: computational model
Mode 1: 299.09 Hz
Mode 2: 1297.9 Hz
Mode 3: 1417.6 Hz
Mode 4: 1679.3 Hz
Mode 5: 3917.6 Hz
Mode 6: 5149.6 Hz
Mode 7: 6538.1 Hz
Mode 8: 7545.1 Hz
Mode 9: 8377.9 Hz
Mode 10: 8933.4 Hz
Mode 11: 12199 Hz
Mode 12: 13198 Hz
Mode 13: 14941 Hz
Mode 14: 17714 Hz
Mode 15: 18072 Hz

E80 Lecture 10.36: Vibration and System ID
Cantilever: experiment

- Sensors: piezoelectric dynamic strain gauges
- Obtain data in time domain and in frequency domain
- Compare to analytical natural frequencies
- Build a low-pass filter to help analyze frequency data
Rocket (final project)

- Mathematical model
  to obtain natural frequencies and modal shapes
- Computational model
  example: SolidWorks model
- Experimental data
  - tap test in lab using impact hammer and accelerometers
  - sensors during flight
• general solution for a free-free beam is:

\[ y(x, t) = \sum_{n=1}^{\infty} \left( A_n \sin \omega_n t + B_n \cos \omega_n t \right) \sin \frac{n\pi x}{L} \]

![Diagram of a free-free beam](image)

\textit{Figure 1: Schematic of a free-free beam}

• calculations of a beam’s cross sectional properties

\[ A = \pi (r_o^2 - r_i^2) \]

\[ I = \frac{1}{4} \pi (r_o^4 - r_i^4) \]


E80 Lecture 10.39: Vibration and System ID
Rocket: mathematical model

- natural frequencies are:

\[ \omega_n = \beta_n^2 \sqrt{\frac{EI}{\rho A}} = (\beta_n L)^2 \sqrt{\frac{EI}{\rho AL^4}} \]

<table>
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<tr>
<th>Boundary Conditions</th>
<th>Frequency Equations</th>
<th>$\beta_1 L$</th>
<th>$\beta_2 L$</th>
<th>$\beta_3 L$</th>
</tr>
</thead>
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<tr>
<td>Pinned-pinned</td>
<td>$\sin \beta L = 0$</td>
<td>3.141</td>
<td>6.282</td>
<td>9.423</td>
</tr>
<tr>
<td>Fixed-free</td>
<td>$\cos \beta L \cosh \beta L + 1 = 0$</td>
<td>1.875</td>
<td>4.694</td>
<td>7.855</td>
</tr>
<tr>
<td>Fixed-pinned (and pinned-free)</td>
<td>$\tan \beta L = \tanh \beta L$</td>
<td>3.927</td>
<td>7.069</td>
<td>10.210</td>
</tr>
<tr>
<td>Fixed-fixed (and free-free)</td>
<td>$\cos \beta L \cosh \beta L = 1$</td>
<td>4.730</td>
<td>7.853</td>
<td>10.996</td>
</tr>
<tr>
<td>Fixed-sliding (and free-sliding)</td>
<td>$\tan \beta L + \tanh \beta L = 1$</td>
<td>2.365</td>
<td>5.498</td>
<td>8.639</td>
</tr>
</tbody>
</table>

Table 1: Natural Frequencies for Single-Span Beams

Rocket: computational model
Mode 1: 0 Hz
Mode 2: 7.0439E-4 Hz
Mode 3: 1.7816E-3 Hz
Mode 4: 11.752 Hz
Mode 5: 11.802 Hz
Mode 6: 62.133 Hz
Mode 7: 62.287 Hz
Mode 8: 111.02 Hz
Mode 9: 111.06 Hz
Mode 10: 114.37 Hz
Mode 11: 154.73 Hz
Mode 12: 155.32 Hz
Mode 13: 257.09 Hz
Mode 14: 266.75 Hz
Mode 15: 273.79 Hz
Rocket: experimental data

In **lab**: tap test with impact hammer
“TapTestFRF.vi”

https://www.youtube.com/watch?v=XkmgMkDKAyU
Rocket: experimental data

During **flight**: get time data from sensors

Post flight: analyze data in frequency domain
compare to tap test results
compare to model predictions
(from analytical model and/or computational model)
Question

Why might we be interested in rocket vibrations?

• To characterize rocket’s natural frequencies and modal shapes

• Avoid dead spots (nodes) to optimize sensor placement

• Design a vibration isolator to minimize vibrations in payload

• Validate your model
Video of flutter

- https://www.youtube.com/watch?v=OhwLojNerMU