Agenda

- Inertial Measurement Unit
  - classic inertial navigators
  - strapdown inertial navigators

- Instrumental
- Algorithm

- We know that we are interested in maximum altitude of the rocket
  - we're interested in predicting that 51 Flight
  - we'll want to measure it during flight

- In First Flight, what did we measure?
  (altitude, actually pressure)
  → altimeter was the instrument we flew

- What did we calculate from these data?
  - accel, velocity

  - were you happy with these results?
  Would you do things differently? (this was
  1-D; might want 3-D)

  (did rocket go straight up? what affects
  rocket trajectory? → later)
· You have usedlocks in to predict trajectory.

Next 2 lectures will focus on how we predict
trajectory (position) and how we experimentally
measure values that allow us to get hold of
vehicle position, velocity + acceleration.

· We expect flight trajectory to be 3-D and
time-varying. Need \( x, y, z, t \) = how?

(Ask)

accels — how many?

Need to fix 3 accels to our rocket

Will that allow us to get \( x, y, z, t \)?

· We need to know the rotation of the rocket "Frame"
   — the local frame — so we can get accel

   (velocity + position as well) in a global frame.

   how to measure rotation? Gyroscope

   how many rotation/gyro's? = 3 gyro's
**INERTIAL MEASUREMENT UNIT (IMU)**

- 3 gyroscopes to measure rotation angles ($\phi_x$, $\phi_y$, $\phi_z$) or rotation rate ($\omega_x$, $\omega_y$, $\omega_z$) for orthogonal axes

- plus 3 accels to measure linear accel ($a_x$, $a_y$, $a_z$) (again, orthogonal axes)

- couple of different ways to implement this:

  **Classic Inertial Navigation**

  - put accels + gyro on a platform

  - mount this platform on gimbals that allow the platform to rotate about all 3 axes

  - each gimbal frame has a motor that can rotate the frame
- How does it work? If the gyros on the platform sense a rotation, they send a signal to the motors and motors rotate the frames to keep the platform at the same orientation.

  => Classic inertial navigation always has a global frame of reference for the accelerations.

- Gyros used to keep platform in one oriental.
- Accel data may be integrated to get velocity + position. (remember gravity)

- Do you think we have this type of system in our E80 rockets? - heavy. sig. need motors + control system to the motors.

Note - what do we do instead?
STRAP DOWN INERTIAL NAVIGATOR

- Mount accel + gyro to vehicle frame (local frame)
- must do transform to get accel measurements from the local frame to the global frame
- once that's done, integrate to get velocity, and integrate again to get position (remember gravity)
- We'll come back to the transform + integrate in a bit. Let's talk about accel + gyro, how they work.

Accelerometers
- basic principle is mass-spring system
- system undergoing accel has force on the mass \( F = ma \)
- the mass undergoes displacement \( x \)

\[ \Rightarrow a = \frac{kx}{m} \]
As you know, the ESP, \( \text{BEAM} \), is a mass-spring system, with the beam acting as a spring.

Also could do piezoelectric accrals

(Vaguely, the crystal lattice deformation leads to an interaction between mechanical and electrical states of the crystal.)

- Relation between mechanical stress and surface charge in a crystal. Acceleration causes a force \( \rightarrow \) stress in crystal \( \rightarrow \) charge on crystal
  \( \rightarrow \) convert charge to voltage using circuitry
  \( \rightarrow \) output is a voltage proportional to accel

Other types as well - see last year's notes

What do we use? MEMS acceler - micro-electro-

mechanical systems

uses principle of mass-spring system, but done really really small
* Basically we have 2 microstructures: 
  - Little beams of polysilicon, 1-10 μm width, 50-100 μm length.
  - Microstructures next to each other have a certain capacitance between them.

  Aside: Capacitor - stores energy.
  - 2 electrical conductors separated by an insulator (dielectric).
  - When powered, charge builds up on the conductors, are positive.

  \[ C = \frac{\epsilon A}{d} \]
  \( C = \text{capacitance}, \ A = \text{plate area}, \ \epsilon = \text{dielectric constant}, \ d = \text{distance between plates} \)

  Physically related to the dielectric material, area of the plate, and distance between the plates.
OK, we're trying to measure accel. How does having tiny beams + capacitance help us?

Recall: beam can be thought of as a spring-mass system \( \Rightarrow \) accel will give a displacement

displacement in the MEMS system causes what? - capacitance change

So if we have circuitry to measure capacitance and change it to voltage, \( \Rightarrow \) we have a accel.

![Diagram of sensor under acceleration](image)

*Figure 5. Simplified View of Sensor Under Acceleration*

beam deflection \( \approx \) femtometers \( 10^{-15} \) m

(will wave my hands on the "on-board circuit" that measures deflected capacitance and outputs voltage)
Basically, we get a voltage output that is proportional to acceleration.

Note: Our accels measure static accel forces.

(What does this mean? Gravity — so remember to take that into account when you use these data)

We have 2 chips — one one-axis accel (ADXL78) along the long axis of the rocket and one dual-axis accel (ADXL320) measuring accel in two orthogonal radial directions.

Errors:

- Bias — can measure by mounting on a gimbaled gyro, will go directly to MEMS Gyros
- Noise — rate gyros — measure angular velocity rather than rotation angle
How do MEMS Gyros work?

Coriolis effect

- Let's be an observer in a rotating system.
  Imagine a mass moving in this system (not parallel to axis of rotation)

\[ \text{w} \quad \rightarrow \text{Coriolis accel} \]

- As the mass moves further from the center, its speed relative to the ground increases.

The rate of increase of the tangential speed, caused by the radial velocity of the rotating system, is the Coriolis accel: \( -2 \, \text{w} \times \overrightarrow{v} \)

\[ \text{radial velocity of the mass} \]
\[ \text{angular velocity} \, \Omega \]

relative to the center of the rotating ref. frame
So, if we move a mass in a rotating frame, we get Coriolis accel (and force). What do we want from this gyro? Rotation rate? How?

\[ \text{Coriolis force} = -2 \mathbf{m} \mathbf{w} \times \mathbf{v} \]

\[ \mathbf{v} \]

Here is rotation rate.

\[ \text{Coriolis accel} = -2 \mathbf{w} \times \mathbf{v} \]

Any ideas?

Put an oscillating mass-spring system in a rotating frame \( \Rightarrow \) cause Coriolis accel.

Measurable accel as 69 (tingeo + capacitance).

Figure 4. Schematic of the gyro's mechanical structure.


Another we measure a displacement to get hold of accel, and in this case, angular velocity (proportional to voltage).
Application

- Wii
- Batteries of insects - not application, but shows principle in insect world
- Aircraft control
- Robotics
- Virtual reality
- Ship stabilization

(Bias + Bias drift - output when angle velocity = 0 is called the bias. This bias can drift over time due to factors such as temperature and stress)

Noise

Our instrument is the IDG300 - Integrated Dual-Axis Gyro. So we have 2 of these so we can get our 3 axes. We get a redundant measurement along each axis of rotation and measurements in the two orthogonal radial directions (same as the linear accel axes)
Let's get back to the Inertial Navigation itself.

Recall we have 3 linear accel measurements.

These accels are fixed to a local (vehicle) frame.

We are after accel (rel, position) in a global frame.

Typical axes:

- $O_x$: local vertical (up)
- $O_y$: local east
- $O_z$: local north

local, fixed to rocket

Global axes:

$O_{Xg}$

$O_{Yg}$

We need 3 rotation angles (or rates) to relate local frame to global frame.

**Definitions**

**Roll**: rotation about long axis.

**Pitch**: imagine an axis from left to right

(forward is along long axis)

rotation about that axis moves nose (or tail) up and down

**Yaw**: imagine top-to-bottom axis. Rotation moves nose right or left.
Any arbitrary rotation can be decomposed into 3 separate rotations of angles $\phi_x$, $\phi_y$, $\phi_z$

(Skipping steps - see last year's notes)

For small angles,

$$
\begin{bmatrix}
U_x \\
U_y \\
U_z
\end{bmatrix} =
\begin{bmatrix}
1 & -\phi_z & \phi_y \\
\phi_z & 1 & -\phi_x \\
-\phi_y & \phi_x & 1
\end{bmatrix}
\begin{bmatrix}
V_{cx} \\
V_{cy} \\
V_{cz}
\end{bmatrix}
$$

How do we get $\phi$s? Also, the rocket continually changes position/orientation. How do we handle that?

We need to get $R$ matrix from measurements we have made.
Recall: \( \mathbf{u}(t) = \mathbf{R}(t) \mathbf{v}(t) \)

\[
\mathbf{u}(t) = \mathbf{v}(t)_{\text{local}} \mathbf{R}(t)_{\text{global}} \mathbf{v}(t)_{\text{local}}
\]

To track the attitude change we must track the change in \( \mathbf{R} \) with time:

\[
\mathbf{R}(t) = \lim_{s \to 0} \frac{\mathbf{R}(t + s t) - \mathbf{R}(t)}{s t}
\]

where \( \mathbf{R}(t + s t) = \mathbf{R}(t) \mathbf{S}(t) \)

where \( \mathbf{S}(t) \) is the rotation matrix which relates the body frame at time \( t \) to the body frame at time \( t + s t \)

\[
\mathbf{S}(t) = \begin{bmatrix}
1 & -s\phi_x & s\phi_y \\
s\phi_x & 1 & -s\phi_z \\
-s\phi_y & s\phi_z & 1
\end{bmatrix} = \mathbf{I} + s\Phi
\]

\( s\phi_x, s\phi_y, s\phi_z \) are small rotations that occur during \( s t \)
So: \( R(t) = \lim_{St \to 0} \left[ \frac{R(t + St) - R(t)}{St} \right] \)

\[
= \lim_{St \to 0} \left[ \frac{R(t) S(t) - R(t)}{St} \right] = \lim_{St \to 0} \left[ R(t) \frac{S(t)}{St} \right] = R(t) \lim_{St \to 0} \frac{S(t)}{St}
\]

\[ \dot{R}(t) = \ddot{R}(t) \dot{\Phi} \]

Recall that \( \dot{\Phi}_x = \omega_x \), \( \dot{\Phi}_y = \omega_y \), \( \dot{\Phi}_z = \omega_z \),

we will have \( w \) given our \( \Phi \)s.

(remember, we're using \( \Phi \) to get \( R \) from our measurements)

Let's define

\[
\mathbf{N} = \dot{\Phi} = \begin{bmatrix} 0 & -w_x & w_y \\ w_x & 0 & -w_z \\ -w_y & w_z & 0 \end{bmatrix}
\]

so \( \dot{R}(t) = R(t) \mathbf{N}(t) \)

\[
\frac{dR}{dt} = R \dot{R} \quad \text{or} \quad \frac{dR}{R} = \mathbf{N} \, dt
\]
D.E. soln is \( R(t) = R(0) \cdot \exp \left( \int_0^t \omega(t) \, dt \right) \)

where \( R(0) \) is the initial attitude of the device.

[Note: this matrix math]

IMU gives us samples of angular velocity, not a continuous signal. I will present a method to integrate the sampled signal.

For small \( \Delta t \)

\[
\int_{t}^{t+\Delta t} \omega \, dt \approx R'(t) \, dt = B
\]

\( (\text{rectangular rule}) \)

\[
B = \begin{bmatrix}
0 & -w_z \Delta t & w_y \Delta t \\
-w_z \Delta t & 0 & -w_x \Delta t \\
w_y \Delta t & w_x \Delta t & 0
\end{bmatrix}
\]

So \( R(t+\Delta t) = R(t) \cdot \exp B \)
(\(w_x, w_y, w_z\)) are the angular velocity samples corresponding to the update period.

Let \(\Omega^2 = (w_x^2 + w_y^2 + w_z^2)\), do Taylor series expansion of the exponential (see Wargi's lecture '08): 

\[ R(t + \Delta t) = R(t) \left( I + \frac{\sin \Omega \Delta t}{\Omega} + \frac{1 - \cos \Omega \Delta t}{\Omega^2} \right) \]

every time we get new \(w\)s, we update the rotation matrix; this allows us to get the variable in the global frame, \([\text{global} = R \cdot \text{local}]\)

Then integrate accel to get velocity; integrate again to get position.

ERRORS - accel + gyro measurements have noise + bias AND we're integrating these data over + over in position space!
Can lead to large errors over time.
ACCEL + GYRO (IMU) LAB

need to calibrate accels + gyros

- need to troubleshoot IMU's (some IMUs have
  broken accels and/or gyros)

- calculate the change in global position using IMU

data \Rightarrow error? explain

You are given a turntable, IMU, R-DAS

Up to team to come up w/ Design of Experiments and
a plan to do the lab.